SIMPLE — Untyped

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Abstract

This is the 𝕂 semantic definition of the untyped SIMPLE language. SIMPLE is intended to be a pedagogical and research language that captures the essence of the imperative programming paradigm, extended with several features often encountered in imperative programming languages. A program consists of a set of global variable declarations and function definitions. Like in C, function definitions cannot be nested and each program must have one function called main, which is invoked when the program is executed. To make it more interesting and to highlight some of 𝕂's strengths, SIMPLE includes the following features in addition to the conventional imperative expression and statement constructs:

- Multidimensional arrays and array references. An array evaluates to an array reference, which is a special value holding a location (where the elements of the array start) together with the size of the array; the elements of the array can be array references themselves (particularly when the array is multi-dimensional). Array references are ordinary values, so they can be assigned to variables and passed/received by functions.
- Functions and function values. Functions can have zero or more parameters and can return abruptly using a return statement. SIMPLE follows a call-by-value parameter passing style, with static scoping. Function names evaluate to function abstractions, which hereby become ordinary values in the language, same like the array references.
- Blocks with locals. SIMPLE variables can be declared anywhere, their scope being from the place they are declared until the end of the most nested enclosing block.
- Input/Output. The expression read() evaluates to the next value in the input buffer, and the statement write(e) evaluates e and outputs its value to the output buffer. The input and output buffers are lists of values.
- Exceptions. SIMPLE has parametric exceptions (the value thrown as an exception can be caught and bound).
- Concurrency via dynamic thread creation/termination and synchronization. One can spawn a thread to execute any statement. The spawned thread shares with its parent its environment at creation time. Threads can be synchronized via reentrant locks which can be acquired and released, as well as through rendezvous.

Like in many other languages, some of SIMPLE's constructs can be desugared into a smaller set of basic constructs. We do that at the end of the syntax module, and then we only give semantics to the core constructs.

Note: This definition is commented slightly more than others, because it is intended to be one of the first that the new user of 𝕂 sees. For a quick introduction to the K prototype, you are referred to the README file at the root of this k-framework distribution. If you are interested in reading more about 𝕂, please check the following paper:

Grigore Roșu, Traian Florin Șerbănuță: An overview of the K semantic framework. Journal of Logic and Algebraic Programming, 79(6): 397-434 (2010)

MODULE SIMPLE-UNTYPED-SYNTAX

Syntax

We start by defining the SIMPLE syntax. The language constructs discussed above have the expected syntax and evaluation strategies. Recall that in 𝕂 we annotate the syntax with appropriate strictness attributes, thus giving each language construct the desired evaluation strategy.

Identifiers

The special identifier for the function "main" belongs to all programs. Each program may use additional identifiers, which need to be declared either automatically (when one uses an external parser) or manually (when one writes the program).

SYNTAX #Id ::= main

Declarations

There are two types of declarations: for variables (including arrays) and for functions.

```
SYNTAX  Ids  ::= List{#Id,","}
SYNTAX  Exps ::= List{Exp,","} [strict]
SYNTAX  Decl ::= var Exps ;
               | function #Id ( Ids ) Stmt
```

Expressions

The expression constructs below are standard. Increment (++) takes an expression rather than a variable because it can also increment an array element. Arrays can be multidimensional and can hold other arrays, so their lookup operation takes a list of expressions as argument and applies to an expression (which can in particular be another array lookup), respectively. The construct sizeOf gives the size of an array in number of elements of its first dimension. Note that almost all constructs are strict. Exceptions are the increment (since its first argument gets updated, so it cannot be evaluated) and the assignment which is only strict in its second argument (for the same reason as the increment).

```
SYNTAX  Exp ::= #Int
              | #Bool
              | #String
              | #Id
              | ++ Exp
              | Exp + Exp [strict]
              | Exp - Exp [strict]
              | Exp * Exp [strict]
              | Exp / Exp [strict]
              | Exp % Exp [strict]
              | - Exp [strict]
              | Exp < Exp [strict]
              | Exp <= Exp [strict]
              | Exp > Exp [strict]
              | Exp >= Exp [strict]
              | Exp == Exp [strict]
              | Exp != Exp [strict]
              | Exp and Exp [strict]
              | Exp or Exp [strict]
              | not Exp [strict]
              | Exp [ Exps ] [strict]
              | sizeOf( Exp ) [strict]
              | Exp ( Exps ) [strict]
              | read()
              | Exp = Exp [strict(2)]
```

Statements

Most of the statement constructs are standard for imperative languages. We syntactically distinguish between empty and non-empty blocks, because we chose Stmts not to be a (";"-)separated list of Stmt. Variables can be declared anywhere inside a block, their scope ending with the block. Expressions are allowed to be used for their side effects only (followed by a semicolon ";"). Functions are allowed to abruptly return. The exceptions are parametric, i.e., one can throw a value which is bound to the variable declared by catch. Threads can be dynamically created and terminated, and can synchronize with acquire, release and rendezvous. Note that the strictness attributes obey the intended evaluation strategy of the various constructs. In particular, the if-then-else construct is strict only in its first argument (the if-then construct will be desugared into if-then-else), while the loop constructs are not strict in any arguments. The print statement construct is variadic, that is, it takes an arbitrary number of arguments.

```
SYNTAX  Stmt ::= {}
               | { Stmts }
               | Exp ; [strict]
               | if Exp then Stmt else Stmt [strict(1)]
               | if Exp then Stmt
               | while Exp do Stmt
               | for #Id = Exp to Exp do Stmt
               | return Exp ; [strict]
               | return ;
               | print( Exps ) ; [strict]
               | try Stmt catch( #Id ) Stmt
               | throw Exp ; [strict]
               | spawn Stmt
               | acquire Exp ; [strict]
               | release Exp ; [strict]
               | rendezvous Exp ; [strict]
SYNTAX  Stmts ::= Decl
                | Stmt
                | Stmts Stmts
```

Desugared Syntax

This part desugars some of SIMPLE's language constructs into core ones. We only want to give semantics to core constructs, so we get rid of the derived ones before we start the semantics. All desugaring macros below are straightforward. For the semantics, we can therefore assume that all functions take a list of arguments, that each conditional has both branches, that there are only while loops, and that each variable is declared alone and is initialized.

```
MACRO  if E then S = if E then S else {}
MACRO  for X = E1 to E2 do S = { var X = E1 ; while X <= E2 do { S X = X + 1 ; } }
MACRO  var E1 , E2 , Es ; = var E1 ; var E2 , Es ;
MACRO  var X = E ; = var X ; X = E ;
```

END MODULE

MODULE SIMPLE-UNTYPED
IMPORTS SIMPLE-UNTYPED-SYNTAX
SYNTAX Vals ::= List{Val,","}

Basic Semantic Infrastructure

Before one starts adding semantic rules to a 𝕂 definition, one needs to define the basic semantic infrastructure consisting of definitions for values, computations and the configuration. The values are needed to know when to stop applying the heating rules corresponding to strictness or context declarations. As values carry semantic, language dependent meaning, they cannot be automatically inferred, so one cannot avoid this step. The computation structures are needed to properly include the language syntax under the K sort, as the 𝕂 semantics operates only on terms of sort K; fortunately, this step can be done mostly automatically, except for the special result computations. Finally, the configuration serves as a backbone for the process of configuration abstraction which allows users to only mention the relevant cells in each semantic rule, the rest of the configuration context being inferred automatically. Although the configuration could potentially be automatically inferred from the rules, we believe that it is very useful for language designers/semanticists to actually think of and design their configuration explicitly, so the current implementation of 𝕂 requires one to define it.

Values

We here define the values of the language that the various fragments of programs evaluate to. First, integers and Booleans are values. As arrays, they evaluate to special array reference values holding (1) a location from where the array's elements are contiguously allocated in the store, and (2) the size of the array. Functions evaluate to function values as λ-abstractions (we do not need to evaluate functions to closures because each function is executed in the fixed global environment and function definitions cannot be nested).

```
SYNTAX  Val ::= #Int
              | #Bool
              | #String
              | array( #Nat , #Nat )
              | λ Ids . Stmt
```

The inclusion of values in expressions follows the methodology of syntactic definitions (like, e.g., in SOS): extend the syntax of the language to encompass all values and additional constructs needed to give semantics. In addition to that, it allows us to write the semantic rules using the original syntax of the language, and to parse them with the same (now extended with additional values) parser. If writing the semantics directly on the 𝕂 AST, using the associated labels instead of the syntactic constructs, then one would not need to include values in expressions.

SYNTAX Exp ::= Val

Computations

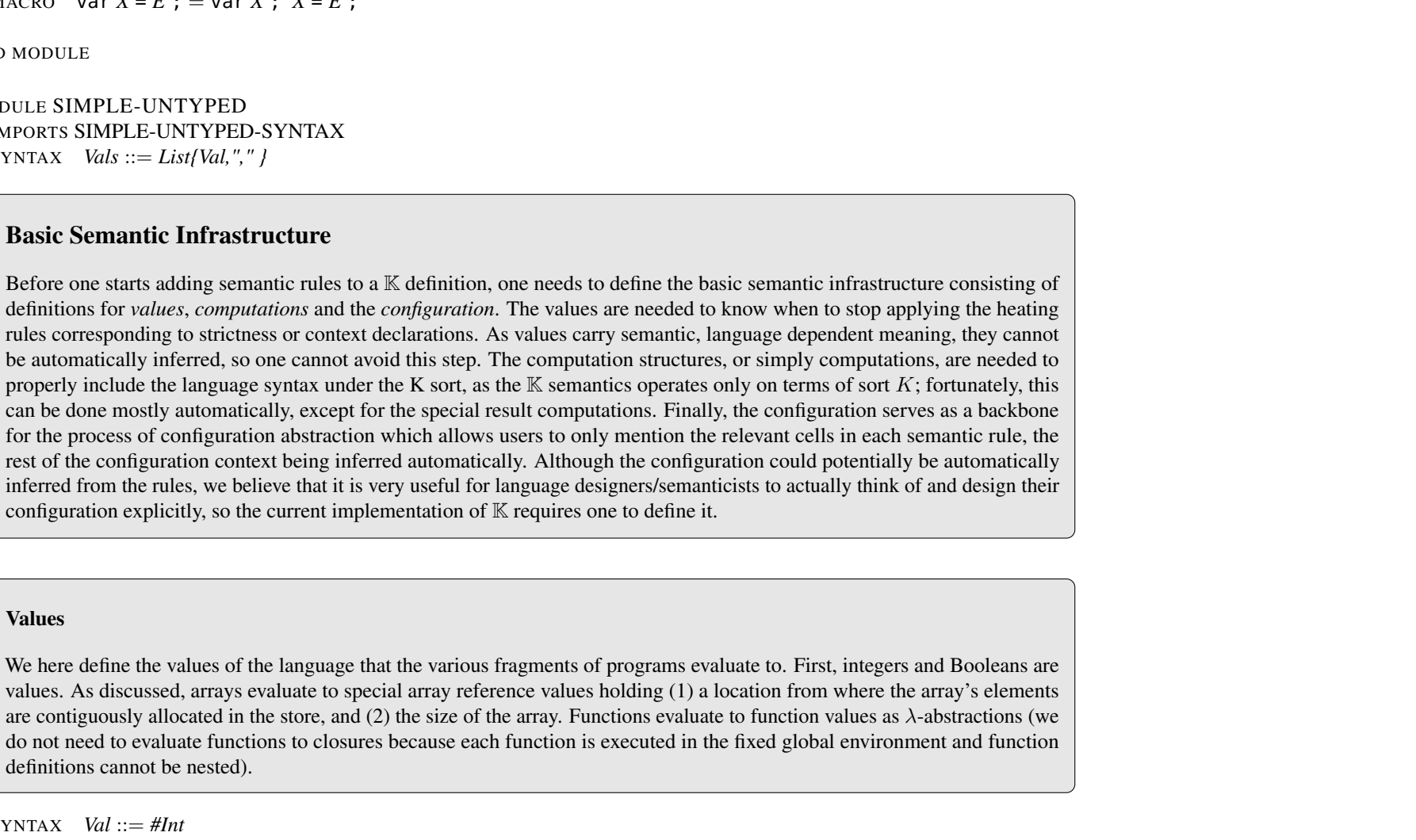
Recall that in 𝕂, the computations "swallow" the entire language syntax (note that the builtin module K is included). Thus, every fragment of program is a particular computation, that is, a term of sort K. Also, recall that computations become, via a simple syntactic abstract syntax trees (where the original language constructs become AST node labels, i.e., constants of sort *KLabel*) extended with the special task sequentialization construct $K \curvearrowright K$. In addition to the sorts K and *KLabel*, the K tool also has one more important sort related to computations, namely *KResult*. Distinguishing terms of sort *KResult* is not necessary in theory, but it is crucial in implementations of the 𝕂 framework, because it allows to distinguish a class of terms as irreducible and thus to more efficiently implement the heating/cooling process associated to the strictness attributes. Below we state that all values are terms of sort *KResult*.

SYNTAX KResult ::= Val

Configuration

The 𝕂 configuration of SIMPLE consists of a top level cell, T, holding a threads cell, a global environment map cell genv mapping the global variables and function names to their locations, a shared store map cell store mapping each location to some value, a set of busy locks busy, two cells in and out holding the input and output lists, as well as a nextLoc cell, holding a natural number indicating the next available location. For simplicity, the location counter in nextLoc models an actual physical location in the store; we assume no garbage collection. In other definitions (such as KERNELC) we show how one can model locations in the store to be symbolic and thus abstract away from the memory allocator library. The threads cell contains one thread cell for each existing thread in the program. Note that the thread cell has multiplicity "*", which means that at any given moment there could be zero, one or more thread cells. Each thread cell contains a computation cell k, a control cell holding the various control structures needed to jump to certain points of interest in the program execution, a local environment map cell env mapping the thread local variables to locations, and a holds cell holding the locks that the thread holds and has not yet released and how many times it has acquired them. The control cell currently contains only two subcells, a function stack fstack which is a list and an exception stack xstack which is also a list. One can add more control structures in the control cell, such as a stack for break/continue of loops, etc., if the language is extended with more control-changing constructs. Note that all cells except for k are also initialized, in that they contain a ground term of their corresponding sort. The k cell is initialized with the program that will be passed to the K tool, as indicated by the $PGM variable.

CONFIGURATION:



Declarations and Initialization

We start by defining the semantics of declarations (for variables, arrays and functions).

Variable Declaration

The SIMPLE syntax was desugared above so that each variable is declared alone and its initialization is done as a separate statement. The semantic rule below matches resulting variable declarations of the form "var X;" on top of the k cell (indeed, note that the k cell is complete, or round, to the left, and is torn, or ruptured, to the right), allocates a fresh location L in the store which is initialized with a special value ⊥ (indeed, the unit "•", or nothing, is matched anywhere in the map—note the torn sides—and replaced with the mapping $L \mapsto \bot$), and binds X to L in the local environment shadowing previous declarations of X, if any. This possible shadowing of X requires us to use a similar technique for updating the environment as for updating the store, so we know that X is not already bound in the environment when we add $X \mapsto L$. We prefer to use the map update operation on the environment instead, because the environment is frozen once it is passed to a closure. Also, notice that the fresh location counter is incremented. All the above happen in one transactional step, with the rule below. Note also how configuration abstraction allows us to only mention the needed cells; indeed, as the configuration above states, the k and env cells are actually located within a thread cell within the threads cell, but one need not mention these: the configuration context of the rule is automatically transformed to match the declared configuration structure.

Note: The "trick" with using a nextLoc cell to generate fresh locations is rather low level and hopefully temporary; we intend to soon allow instead a side-condition of the form "where L fresh".

SYNTAX K ::= ⊥

RULE ⟨ var X ; ⇒ • ⋯⟩k ⟨ Env ⇒ Env[L / X] ⟩env ⟨⋯ • ⇒ L ↦ ⊥ ⋯⟩store ⟨ L ⇒ L +Nat 1 ⟩nextLoc

Array Declaration

The 𝕂 semantics of the uni-dimensional array declaration is somehow similar to the above declaration of ordinary variables. First, N + 1 locations are allocated in the store for an array of size N, the additional location (chosen to be the first one allocated) holding the array reference value. The array reference value array(L, N) states that the array has size N and its elements are located contiguously in the store starting with location L. Recall that L..L' is the list of locations between L and L' and that L..L' ↦ V initializes each location in the list L..L' to V. Note that, since the dimensions of array declarations can be arbitrary expressions, this virtually means that we can dynamically allocate memory in SIMPLE by means of array declarations.

RULE ⟨ var X [N] ; ⇒ • ⋯⟩k ⟨ Env ⇒ Env[L / X] ⟩env ⟨⋯ • ⇒ L ↦ array(L +Nat 1 , N) L +Nat 1 .. N +Nat L ↦ ⊥ ⋯⟩store ⟨ L ⇒ N +Nat L +Nat 1 ⟩nextLoc

SIMPLE allows multi-dimensional arrays. For semantic simplicity, we desugar them all into uni-dimensional arrays by code transformation. This way, we only need to give semantics to uni-dimensional arrays. First, the context rule below allows us to request the evaluation of the array dimensions:

CONTEXT: var X [□] ;

Upon evaluating the array dimensions, the code generation rule below desugars multi-dimensional array declaration to uni-dimensional declarations. To this aim, we introduce two special unique variable identifiers, $1 and $2. The first, $1, is assigned the array reference value of the current array, so that we can redeclare the array inside the loop body with fewer dimensions. The second variable, $2, iterates through and initializes each element of the current dimension:

```
SYNTAX  #Id ::= $1
              | $2
RULE  var X [ N1 , N2 , Vs ] ; ⇒ var X [ N1 ] ; { var $1 = X ; for $2 = 0 to N1 -Int 1 do { var X [ N2 , Vs ] ; $1 [ $2 ] = X ; } }   [structural]
```

Ideally, one would like to perform syntactic desugarings like the one above before the actual semantics. Unfortunately, that was not possible in this case because the dimension expressions of the multi-dimensional array need to be evaluated first. Indeed, the desugaring rule above does not work if the dimensions of the declared array are arbitrary expressions, because they can have side effects (e.g., a[++x,++x]) and those side effects would be propagated each time the expression is evaluated in the desugaring code (note that both the loop condition and the nested multi-dimensional declaration would need to evaluate the expressions given as array dimensions).

Function declaration

Functions are evaluated to λ-abstractions and stored like any other values in the store. A binding is added into the environment for the function name to the location holding its body. Similarly to the C language, SIMPLE only allows function declarations at the top level of the program. More precisely, the subsequent semantics of SIMPLE only works well when one respects this requirement. Indeed, the simplistic context-free parser generated by the grammar above is more generous than we may want, in that it allows function declarations anywhere any declaration is allowed, including inside arbitrary blocks. However, as the rule below shows, we are not storing the declaration environment with the λ-abstraction value as closures do. Instead, as seen shortly, we switch to the global environment whenever functions are invoked, which is consistent with our requirement that functions should only be declared at the top. Thus, if one declares local functions, then one may see unexpected behaviors (e.g., when one shadows a global variable before declaring a local function). The type checker of SIMPLE, also defined in 𝕂 (see simple/typed/static), discards programs which do not respect this requirement.

RULE ⟨ function F (Xs) S ⇒ • ⋯⟩k ⟨ Env ⇒ Env[L / F] ⟩env ⟨⋯ • ⇒ L ↦ λXs . S ⋯⟩store ⟨ L ⇒ L +Nat 1 ⟩nextLoc

When we are done with the first pass (pre-processing), the computation cell k contains only the token execute (see the configuration declaration above, where the computation item execute is placed right after the program in the k cell) and the cell genv is empty. In this case, we have to call main() and to initialize the global environment by transferring the contents of the local environment into it. We prefer to do it this way, as opposed to processing all the top level declarations directly within the global environment, because we want to avoid duplication of semantics: the syntax of the global declarations is identical to that of their corresponding local declarations, so the semantics of the latter suffices provided that we copy the local environment into the global one once we are done with the pre-processing. We want this separate pre-processing step precisely because we want to create the global environment. All (top-level) functions end up having their names bound in the global environment and, as seen below, they are executed in that same global environment; all these mean, in particular, that the functions "see" each other, allowing for mutual recursion, etc.

SYNTAX K ::= execute

RULE ⟨ execute ⇒ main(•) ; ⟩k ⟨ Env ⟩env ⟨ • ⇒ Env ⟩genv

Expressions

We next define the 𝕂 semantics of all the expression constructs, in the order in which their syntax was declared.

Variable lookup

When a variable X is the first computational task, and X is bound to some location L in the environment (note the rupture of the env cell at both sides), and L is mapped to some value V in the store, then we rewrite X by V:

RULE ⟨ X ⇒ V ⋯⟩k ⟨⋯ X ↦ L ⋯⟩env ⟨⋯ L ↦ V ⋯⟩store   [transition]

Note that this excludes reading ⊥, as ⊥ is not a value.

Variable/Array increment

This is tricky, because we want to allow both ++x and ++a[5]. Therefore, we need to extract the l-value of the expression to increment. To do that, we state that the expression to increment should be wrapped by the auxiliary "l-value" operation and then evaluated. The semantics of this auxiliary l-value operation is defined below. For now, all we need to know is that it takes an expression and evaluates to a location value, also introduced below with the auxiliary operations.

CONTEXT: ++ (□ ⇒ l-value(□))

RULE ⟨ ++ loc(L) ⇒ I +Int 1 ⋯⟩k ⟨⋯ L ↦ (I ⇒ I +Int 1) ⋯⟩store   [transition]

Arithmetic operators

There is nothing special about the following rules. They rewrite the language constructs to their library counterparts when their arguments become values of expected sorts:

```
RULE  I1 + I2 ⇒ I1 +Int I2
RULE  Str1 + Str2 ⇒ Str1 +String Str2
RULE  I1 - I2 ⇒ I1 -Int I2
RULE  I1 * I2 ⇒ I1 *Int I2
RULE  I1 / I2 ⇒ I1 /Int I2     when I2 =/=Bool 0
RULE  I1 % I2 ⇒ I1 %Int I2     when I2 =/=Bool 0
RULE  - I ⇒ -Int I
RULE  I1 < I2 ⇒ I1 <Int I2
RULE  I1 <= I2 ⇒ I1 ≤Int I2
RULE  I1 > I2 ⇒ I1 >Int I2
RULE  I1 >= I2 ⇒ I1 ≥Int I2
RULE  V1 == V2 ⇒ V1 ==Bool V2
RULE  V1 != V2 ⇒ V1 =/=Bool V2
RULE  B1 and B2 ⇒ B1 ∧Bool B2
RULE  B1 or B2 ⇒ B1 ∨Bool B2
RULE  not B ⇒ ¬Bool B
```

Array lookup

Untyped SIMPLE does not check array bounds (the dynamically typed version of it, in ../typed/dynamic, does check for array out of bounds). The first rule below desugars the multi-dimensional array access syntax to uni-dimensional array access; recall that the array access operation was declared strict, so all sub-expressions involved are already values at this stage. The second rule rewrites the array access to a lookup operation at a precise location; we prefer to do it this way to avoid locking the store. Recall that "_" is an anonymous variable in 𝕂 (like in Prolog) matching any subterm; informally, "there is something there but we don't care what". The semantics of the lookup operation is straightforward.

```
RULE  V [ N1 , N2 , Vs ] ⇒ V [ N1 ] [ N2 , Vs ]   [structural]
RULE  array( L , _ ) [ N ] ⇒ lookup( L +Nat N )   [structural]
SYNTAX  K ::= lookup( #Nat )
```

RULE ⟨ lookup(L) ⇒ V ⋯⟩k ⟨⋯ L ↦ V ⋯⟩store   [transition]

Size of an array

The size of the array is stored in the array reference value, and the sizeOf construct was declared strict, so:

RULE sizeOf(array(_ , N)) ⇒ N

Function call

Function application was strict in both its arguments, so we can assume that both the function and its arguments are evaluated to values (the former expected to be a λ-abstraction). The first rule below matches a well-formed function application on top of the computation and performs the following steps atomically: it switches to the function body followed by "return;" (for the case in which the function does not use an explicit return statement); it pushes the remaining computation, the current environment, and the current control data onto the function stack (the remaining computation can thus only be discarded from the k cell, but it will always be recovered from the stack); it switches the current environment (which is being pushed on the function stack) to the global environment, which is where the free variables in the function body should be looked up; it binds the formal parameters to fresh locations in the new environment, and stores the actual arguments to those locations in the store. The second rule pops the computation, the environment and the control data from the function stack when a return statement is encountered as the next computational task, passing the returned value to the popped computation (the popped computation was the context in which the returning function was called). Note that the pushing/popping of the control data is crucial. Without it, one may have a function that contains an exception block with a return statement inside, which would put the xstack cell in an inconsistent state (since the exception block modifies it, but that modification should be irrelevant once the function returns). We add an artificial nothing value to the language, which is returned by the nulary return; statements.

SYNTAX ListItem ::= (Map , K , Bag)

RULE ⟨ λXs . S (Vs) ↷ K ⇒ bindto(Xs , Vs) ↷ S ↷ return; ⟩k ⟨ ⟨ • ⇒ (Env , K , C) ⋯⟩fstack C ⟩control ⟨ Env ⇒ GEnv ⟩env ⟨ GEnv ⟩genv

RULE ⟨ return V ; ↷ _ ⇒ V ↷ K ⟩k ⟨ ⟨ (Env , K , C) ⇒ • ⋯⟩fstack _ ⇒ C ⟩control ⟨ _ ⇒ Env ⟩env   [transition]

```
SYNTAX  Val ::= nothing
RULE  return; ⇒ return nothing ;   [structural]
```

The bindto auxiliary construct takes a list of variables to a list of values. Specifically, it allocates a fresh location for each variable, binding the variable to that location in the environment and writing the value to that location in the store.

SYNTAX K ::= bindto(Ids , Vals)

RULE ⟨ bindto((X , Xs) , (V , Vs)) ⇒ bindto(Xs , Vs) ⋯⟩k ⟨ Env ⇒ Env[L / X] ⟩env ⟨⋯ • ⇒ L ↦ V ⋯⟩store ⟨ L ⇒ L +Nat 1 ⟩nextLoc

RULE bindto(• , •) ⇒ •   [structural]

Read

The read() expression construct simply evaluates to the next input value, at the same time discarding the input value from the in cell.

RULE ⟨ read() ⇒ I ⋯⟩k ⟨ I ⇒ • ⋯⟩in   [transition]

Assignment

In SIMPLE, like in C, assignments are expression constructs and not statement constructs. To make it a statement all one needs to do is to follow it by a semi-colon ";" (see the semantics for expression statements below). Like for the increment, we want to allow assignments not only to variables but also to array elements, e.g., e1[e2] = e3 where e1 evaluates to an array reference, e2 to a natural number, and e3 to any value. Thus, we first compute the l-value of the left-hand-side expression that appears in an assignment. Like for the increment, all we need to know is that l-value(...) eventually evaluates to a location value loc(...).

CONTEXT: (□ ⇒ l-value(□)) = _

RULE ⟨ loc(L) = V ⇒ V ⋯⟩k ⟨⋯ L ↦ (_ ⇒ V) ⋯⟩store   [transition]

Statements

We next define the 𝕂 semantics of statements, also in the order their syntax was given.

Blocks

Empty blocks are simply discarded, as shown in the first rule below. For non-empty blocks, we schedule the enclosed statement but we have to make sure the environment is recovered after the enclosed statement executes. Recall that we allow local variable declarations, whose scope is the block enclosing them. That is the reason for which we have to recover the environment after the block. This allows us to have a very simple semantics for variable declarations, as we did above. One can make the two rules below computational if one wants them to count as computational steps.

RULE {} ⇒ •   [structural]

RULE ⟨ { Ss } ⇒ Ss ↷ env(Env) ⋯⟩k ⟨ Env ⟩env   [structural]

The basic definition of environment recovery is straightforward:

SYNTAX K ::= env(Map)

RULE ⟨ env(Env) ⇒ • ⋯⟩k ⟨ _ ⇒ Env ⟩env   [structural]

While theoretically sufficient, the basic definition for environment recovery alone is suboptimal. Consider a loop while E do S, whose semantics (see below) is given by unrolling. Typically S is a block. Then the semantics of blocks above, together with the unrolling semantics of the while loop below, will yield a computation structure in the k cell that increasingly grows, adding a new environment recovery task right in front of the already existing sequence of similar environment recovery tasks (this phenomenon is similar to the "tail recursion" problem). Of course, when we have a sequence of environment recovery tasks, we only need to keep the last one. The elegant rule below does precisely that, thus avoiding the unnecessary computation explosion problem:

RULE (env(_) ⇒ •) ↷ env(_)   [structural]

There are two common alternatives to the above semantics of blocks. One is to keep track of the variables which are declared in the block and only recover those at the end of the block. This way one does more work for variable declarations but conceptually less work for environment recovery; we say "conceptually" because it is not clear that it is indeed the case that one does less work when AC matching is involved. The other alternative is to work with a stack of environments instead of a flat environment, and push the current environment when entering a block and pop it when exiting it. This way, one does more work when accessing variables (since one has to search the variable in the environment stack in a top-down manner), but on the other hand uses smaller environments and the definition gets closer to an implementation. Based on experience with dozens of language semantics and other 𝕂 definitions, we have found that our approach above is the best trade-off between elegance and efficiency (especially since rewrite engines have built-in techniques to lazily copy terms, by need, thus not creating unnecessary copies), so it is the one that we follow in general.

Sequential composition

Sequential composition is desugared into 𝕂's builtin sequentialization operation (recall that, like in C, the semi-colon ";" is not a statement separator in SIMPLE—it is either a statement terminator or a construct for a statement from an expression). The rule below is computational, so it does count as a computational step. One can make it structural if one does not want it to count as a step. Note that 𝕂 allows to define the semantics of SIMPLE in such a way that statements eventually dissolve from the top of the computation when they are completed; this is in sharp contrast to (artificially) "evaluating" them to a special skip statement value and then getting rid of that special value, as it is the case in other semantic approaches (where everything must evaluate to something). This means that once S_1 completes in the rule below, S_2 becomes automatically the next computation item without any additional (explicit or implicit) rules.

RULE S_1 S_2 ⇒ $S_1 \curvearrowright S_2$

Expression statements

Expression statements are only used for their side effects, so their result value is simply discarded. Common examples of expression statements are ones of the form ++x;, x=e;, e1[e2]=e3;, etc.

RULE V ; ⇒ •

Conditional

Since the conditional was declared with the strict(1) attribute, we can assume that its first argument will eventually be evaluated. The rules below cover the only two possibilities in which the conditional is allowed to proceed (otherwise the rewriting process gets stuck).

```
RULE  if true then S else _ ⇒ S
RULE  if false then _ else S ⇒ S
```

While loop

The simplest way to give the semantics of the while loop is by unrolling. Note, however, that its unrolling is only allowed when the while loop reaches the top of the computation (to avoid non-termination of unrolling). We prefer the rule below to be structural, because we don't want the unrolling of the while loop to count as a computational step; this is unavoidable in conventional semantics, but it is possible in 𝕂 thanks to its distinction between structural and computational rules. The simple while loop semantics below works because our while loops in SIMPLE are indeed very basic. If we allowed break/continue of loops then we would need a completely different semantics, which would also involve the control cell.

RULE ⟨ while E do S ⇒ if E then { S while E do S } else {} ⋯⟩k   [structural]

Print

The print statement was strict, so all its arguments are now evaluated (recall that print is variadic). We append each of its evaluated arguments to the output buffer, and discard the residual print statement with an empty list of arguments.

RULE ⟨ print(V , Vs); ⇒ print(Vs); ⋯⟩k ⟨⋯ • ⇒ V ⟩out   [transition]

RULE print(); ⇒ •   [structural]

Exceptions

SIMPLE allows parametric exceptions, in that one can throw and catch a particular value. The statement "try S_1 catch(X) S_2" proceeds with the evaluation of S_1. If S_1 evaluates normally, i.e., without any exception thrown, then S_2 is discarded and the execution continues normally. If S_1 throws an exception with a statement of the form "throw E", then E is first evaluated to some value V (throw was declared to be strict), then V is bound to X, then S_2 is evaluated in the new environment while the reminder of S_1 is discarded, then the environment is recovered and the execution continues normally with the statement following the "try S_1 catch(X) S_2" statement. Exceptions can be nested and the statements in the "catch" part (S_2 in our case) can throw exceptions to the upper level. One should be careful with how one handles the control data structures here, so that the abrupt changes of control due to exception throwing and to function returns interact correctly with each other. For example, we want to allow function calls inside the statement S_1 in a "try S_1 catch(X) S_2" block which can throw an exception that is not caught by the function but instead is propagated to the "try S_1 catch(X) S_2" block that called the function. Therefore, we have to make sure that the function stack as well as other potential control structures are also properly modified when the exception is thrown to correctly recover the execution context. This can be easily achieved by pushing/popping the entire current control context onto the exception stack. The three rules below modularly do precisely the above.

```
SYNTAX  ListItem ::= ( #Id , Stmt , K , Map , Bag )
SYNTAX  K ::= popx
```

RULE ⟨ try S_1 catch(X) S_2 ↷ K ⇒ S_1 ↷ popx ⟩k ⟨ ⟨ • ⇒ (X , S_2 , K , Env , C) ⋯⟩xstack C ⟩control ⟨ Env ⟩env

RULE ⟨ popx ⇒ • ⋯⟩k ⟨ _ ⇒ • ⋯⟩xstack

RULE ⟨ throw V ; ↷ _ ⇒ { var X = V ; S_2 } ↷ K ⟩k ⟨ ⟨ (X , S_2 , K , Env , C) ⇒ • ⋯⟩xstack _ ⇒ C ⟩control ⟨ _ ⇒ Env ⟩env

The catch statement S_2 needs to be executed in the original environment, but where the thrown value V is bound to the catch variable X. We here chose to rely on two previously defined constructs when giving semantics to the catch part of the statement: (1) the variable declaration with initialization, for binding X for V; and (2) the block construct for preventing X from shadowing variables in the original environment upon the completion of S_2. Note, however, that the semantics of throw can also be given directly, in one computational step, especially in languages without variable initializers and blocks.

Threads

SIMPLE's threads can be created and terminated dynamically, and can synchronize by acquiring and releasing re-entrant locks and by rendezvous. We discuss the seven rules giving the semantics of these operations below.

Thread creation

Threads can be created by any other threads using the "spawn S" statement. The spawn statement is consumed in the creating thread and, at the same time, a new thread cell is added to the top of the configuration, initialized with the S statement and sharing the same environment with the spawning thread. Note that the newly created thread cell is torn. That means that the remaining cells are added and initialized automatically as described in the definition of SIMPLE's configuration. This is part of 𝕂's configuration abstraction/concretization mechanism.

RULE ⟨⋯ ⟨ spawn S ⇒ • ⋯⟩k ⟨ Env ⟩env ⋯⟩thread (• ⇒ ⟨⋯ ⟨ S ⟩k ⟨ Env ⟩env ⋯⟩thread)

Thread termination

Dually to the above, when a thread terminates its assigned computation (the contents of its k cell) is empty, so the thread can be dissolved. However, since no discipline is imposed on how locks are acquired and released, it can be the case that a terminating thread still holds locks. Those locks must be released, so other threads attempting to acquire them do not deadlock. We achieve that by removing all the locks held by the terminating thread in its holds cell from the set of busy locks in the busy cell (keys(H) returns the domain of the map H as a set, that is, only the locks themselves ignoring their multiplicity). As seen below, a lock is added to the busy cell as soon as it is acquired for the first time by a thread.

RULE (⟨⋯ ⟨ • ⟩k ⟨ H ⟩holds ⋯⟩thread ⇒ •) ⟨ Busy ⇒ Busy -Set keys H ⟩busy

Acquire lock

There are two cases to distinguish when a thread attempts to acquire a lock (in SIMPLE any value can be used as a lock): (1) The thread does not currently have the lock, in which case it has to take it provided that the lock is not already taken by another thread (see the side condition of the first rule). (2) The thread already has the lock, in which case it just increments its counter for the lock (the locks are re-entrant). These two cases are captured by the two rules below:

RULE ⟨ acquire V ; ⇒ • ⋯⟩k ⟨⋯ • ⇒ V ↦ 0 ⋯⟩holds ⟨ Busy (• ⇒ V) ⟩busy   when ¬Bool V in Busy   [transition]

RULE ⟨ acquire V ; ⇒ • ⋯⟩k ⟨⋯ V ↦ (N ⇒ N +Nat 1) ⋯⟩holds

Release lock

Similarly, there are two corresponding cases to distinguish when a thread releases a lock: (1) The thread holds the lock more than once, in which case all it needs to do is to decrement the lock counter. (2) The thread holds the lock only once, in which case it needs to remove it from its holds cell and also from the the shared busy cell, so other threads can acquire it if they need to.

RULE ⟨ release V ; ⇒ • ⋯⟩k ⟨⋯ V ↦ (N ⇒ N -Int 1) ⋯⟩holds   when N >Nat 0

RULE ⟨ release V ; ⇒ • ⋯⟩k ⟨⋯ V ↦ 0 ⇒ • ⋯⟩holds ⟨⋯ V ⇒ • ⋯⟩busy

Rendezvous synchronization

In addition to synchronization through acquire and release of locks, SIMPLE also provides a construct for rendezvous synchronization. A thread whose next statement to execute is rendezvous(V) gets stuck until another thread reaches an identical statement; when that happens, the two threads drop their rendezvous statements and continue their executions. If three threads happen to have an identical rendezvous statement as their next statement, then precisely two of them will synchronize and the other will remain blocked until another thread reaches a similar rendezvous statement. The rule below is as simple as it can be. Note, however, that, again, it is 𝕂's mechanism for configuration abstraction that makes it work as desired: since the only cell which can multiply containing a k cell inside is the thread cell, the only way to concretize the rule below to the actual configuration of SIMPLE is to include each k cell in a thread cell.

RULE ⟨ rendezvous V ; ⇒ • ⋯⟩k ⟨ rendezvous V ; ⇒ • ⋯⟩k   [transition]

Auxiliary declarations and operations

l-value and loc

For convenience in giving the semantics of constructs like the increment and the assignment, that we want to operate the same way on variables and on array elements, we used an auxiliary l-value(E) construct which was expected to evaluate to the l-value of the expression E. This is only defined when E has an l-value, that is, when E is either a variable or evaluates to an array element. l-value(E) evaluates to a value of the form loc(L), where L is the location where the value of E can be found; for clarity, we use loc to structurally distinguish natural numbers from location values. In giving semantics to l-value there are two cases to consider. (1) If E is a variable, then all we need to do is to grab its location from the environment. (2) If E is an array element, then we first evaluate the array and its index in order to identify the exact location of the element of concern, and then return that location; the last rule below works because its preceding context declarations ensure that the array and its index are evaluated, and then the rule for array lookup (defined above) rewrites the evaluated array access construct to its corresponding store lookup operation.

```
SYNTAX  Exp ::= l-value( K )
SYNTAX  Val ::= loc( #Nat )
```

RULE ⟨ l-value(X) ⇒ loc(L) ⋯⟩k ⟨⋯ X ↦ L ⋯⟩env

CONTEXT: l-value(_ [□])

CONTEXT: l-value(□ [_])

RULE ⟨ l-value(lookup(L)) ⇒ loc(L) ⋯⟩k

Sequences of locations

The following operation expands to the list of natural numbers between two given numbers. The first number is expected to be no larger than the second. The two rules below are structural, for the same reason as above.

```
SYNTAX  List{K} ::= #Nat .. #Nat
RULE  N1 .. N2 ⇒ •     when N1 >Nat N2                          [structural]
RULE  N1 .. N2 ⇒ N1 , N1 +Nat 1 .. N2     when N1 ≤Nat N2        [structural]
```

The semantics of SIMPLE is now complete.

END MODULE